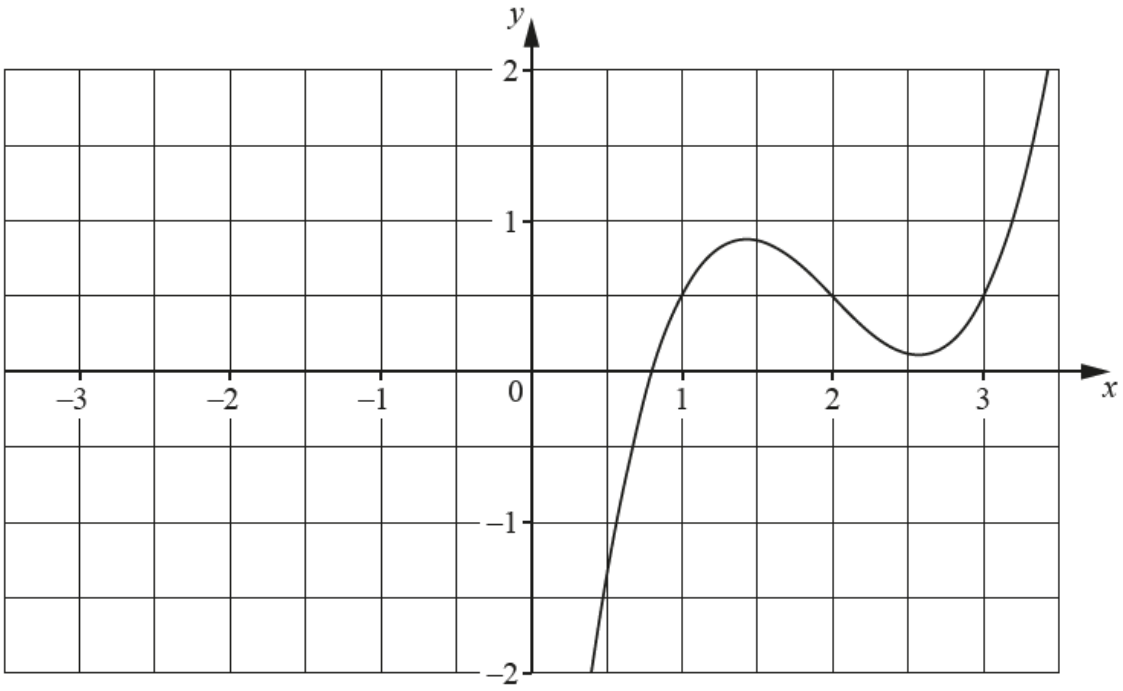
Algebra and Functions – 2021/20 GCE AS Mathematics A

1. Oct/2021/Paper_H230/01/No.3

The diagram shows the curve $y = f(x)$, where $f(x)$ is a cubic polynomial in x.
This diagram is repeated in the Printed Answer Booklet.



(a) State the values of x for which $f(x) < \frac{1}{2}$, giving your answer in set notation. **[3]**

(b) On the diagram in the Printed Answer Booklet, draw the graph of $y = f(-x)$. **[2]**

(c) Explain how you can tell that $f(x)$ cannot be expressed as the product of three real linear factors. **[1]**

2. Oct/2021/Paper_H230/01/No.6

The power output, P watts, of a certain wind turbine is proportional to the cube of the wind speed $v\,\text{ms}^{-1}$.

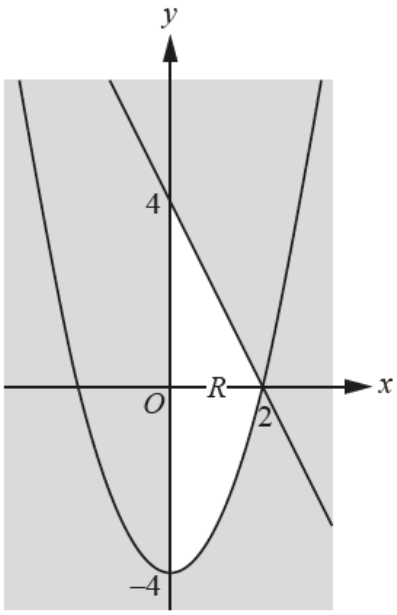
When $v = 3.6$, $P = 50$.

Determine the wind speed that will give a power output of 225 watts. **[3]**

3. Oct/2021/Paper_H230/02/No.1

Given that $(x-2)$ is a factor of $2x^3 + kx - 4$, find the value of the constant k. **[2]**

4. Oct/2021/Paper_H230/02/No.2



The diagram shows the line $y = -2x+4$ and the curve $y = x^2 - 4$. The region R is the unshaded region together with its boundaries.

Write down the inequalities that define R. **[3]**

5. Oct/2020/Paper_H230/01/No.5

The function f is defined by $f(x) = (x+a)(x+3a)(x-b)$ where a and b are positive integers.

(a) On the axes in the Printed Answer Booklet, sketch the curve $y = f(x)$. **[2]**

(b) On your sketch show, in terms of a and b, the coordinates of the points where the curve meets the axes. **[2]**

It is now given that $a = 1$ and $b = 4$.

(c) Find the total area enclosed between the curve $y = f(x)$ and the x-axis. **[4]**

6. Oct/2020/Paper_H230/01/No.6

In this question you must show detailed reasoning.

(a) Solve the inequality $x^2 + x - 6 > 0$, giving your answer in set notation. **[4]**

(b) Solve the equation $x^3 - 7x^{\frac{3}{2}} - 8 = 0$. **[4]**

(c) Find the exact solution of the equation $(3^x)^2 = 3 \times 2^x$. **[5]**

7. Oct/2020/Paper_H230/01/No.7

Determine the points of intersection of the curve $3xy + x^2 + 14 = 0$ and the line $x + 2y = 4$. **[5]**

8. Oct/2020/Paper_H230/02/No.5

A curve has equation $y = a(x+b)^2 + c$, where a, b and c are constants. The curve has a stationary point at $(-3, 2)$.

(a) State the values of b and c. **[2]**

When the curve is translated by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ the transformed curve passes through the point $(3, -18)$.

(b) Determine the value of a. **[3]**

9. Oct/2020/Paper_H230/02/No.4

The cubic polynomial $6x^3 + kx^2 + 57x - 20$ is denoted by $\mathrm{f}(x)$. It is given that $(2x-1)$ is a factor of $\mathrm{f}(x)$.

(a) Use the factor theorem to show that $k = -37$. **[2]**

(b) Using this value of k, factorise $\mathrm{f}(x)$ completely. **[3]**

(c) **(i)** Hence find the three values of t satisfying the equation $6\mathrm{e}^{-3t} - 37\mathrm{e}^{-2t} + 57\mathrm{e}^{-t} - 20 = 0$. **[2]**

(ii) Express the sum of the three values found in part **(c)(i)** as a single logarithm. **[2]**

10. Oct/2020/Paper_H230/02/No.2

Two curves have equations $y = \ln x$ and $y = \frac{k}{x}$, where k is a positive constant.

(a) Sketch the curves on a **single** diagram. **[3]**

(b) Explain how your diagram shows that the equation $x \ln x - k = 0$ has exactly one real root. **[2]**

11. June/2019/Paper_H230/01/No.3(a)

In this question you must show detailed reasoning.

(a) The polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 8x + 3$.

(i) Show that $f(1) = 0$. **[1]**

(ii) Solve the equation $f(x) = 0$. **[4]**

12. June/2019/Paper_H230/02/No.1

In this question you must show detailed reasoning.

Solve the equation $x(3-\sqrt{5})=24$, giving your answer in the form $a+b\sqrt{5}$, where a and b are positive integers. **[3]**

13. June/2019/Paper_H230/02/No.3

(a) Sketch the curve $y=-\frac{1}{x^2}$. **[1]**

(b) The curve $y=-\frac{1}{x^2}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated. **[2]**

(c) The curve $y=-\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor $\frac{1}{2}$ and, as a result, the point $\left(\frac{1}{2}, -4\right)$ on the curve is transformed to the point P.

State the coordinates of P. **[2]**

14. June/2019/Paper_H230/02/No.8

(a) Show that the equation $2\log_2 x = \log_2(kx-1)+3$, where k is a constant, can be expressed in the form $x^2-8kx+8=0$. **[4]**

(b) Given that the equation $2\log_2 x = \log_2(kx-1)+3$ has only one real root, find the value of this root. **[4]**