Algebra and Functions – 2021/20 GCE Pure Mathematics A

1. Nov/2021/Paper_H240/01/No.1

   Determine the set of values of k such that the equation $x^2 + 4x + (k+3) = 0$ has **two** distinct real roots. [4]

2. Nov/2021/Paper_H240/01/No.2

   Alex is comparing the cost of mobile phone contracts. Contract A has a set-up cost of £40 and then costs 4p per minute. Contract B has no set-up cost, does not charge for the first 100 minutes and then costs 6p per minute.

   (a) Find an expression for the cost of each of the contracts in terms of m, where m is the number of minutes for which the phone is used and $m > 100$. [2]

   (b) Hence find the value of m for which both contracts would cost the same. [2]

3. Nov/2021/Paper_H240/01/No.3

   It is given that x is proportional to the product of the square of y and the positive square root of z. When $y = 2$ and $z = 9$, $x = 30$.

   (a) Write an equation for x in terms of y and z. [2]

   (b) Find the value of x when $y = 3$ and $z = 25$. [2]

4. Nov/2021/Paper_H240/01/No.4

   In this question you must show detailed reasoning.

   The cubic polynomial $f(x)$ is defined by $f(x) = 2x^3 - 3x^2 - 11x + 6$.

   (a) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. [1]

   (b) Express $f(x)$ in fully factorised form. [3]

   (c) Hence solve the equation $2 \times 8^y - 3 \times 4^y - 11 \times 2^y + 6 = 0$. [3]

5. Nov/2021/Paper_H240/01/No.5(a)

(a) The graph of the function $y = f(x)$ passes through the point P with coordinates $(2, 6)$, and is a one-one function. State the coordinates of the point corresponding to P on each of the following curves.
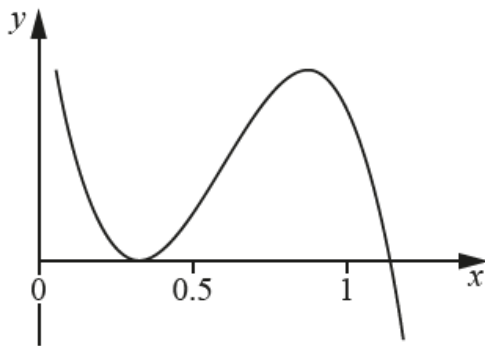
(i) $y = f(x) + 3$ [1]

(ii) $y = 2f(3x - 1)$ [2]

(iii) $y = f^{-1}(x)$ [1]

6. Nov/2021/Paper_H240/02/No.2

The diagram shows part of the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial in x.

Explain why **one** of the roots of the equation $f(x) = 0$ cannot be found by the sign change method. [2]

7.

The size, P, of a population of a certain species of insect at time t months is modelled by the following formula.

$P = 5000 - 1000\cos(30t)°$

(a) Write down the maximum size of the population. **[1]**

(b) Write down the difference between the largest and smallest values of P. **[1]**

(c) Without giving any numerical values, describe briefly the behaviour of the population over time. **[1]**

(d) Find the time taken for the population to return to its initial size for the first time. **[2]**

(e) Determine the time on the second occasion when $P = 4500$. **[4]**

A scientist observes the population over a period of time. He notices that, although the population varies in a way similar to the way predicted by the model, the variations become smaller and smaller over time, and P converges to 5000.

(f) Suggest a change to the model that will take account of this observation. **[1]**

8.

Show in a sketch the region of the x-y plane within which all three of the following inequalities hold.

$y \geqslant x^2, \quad x+y \leqslant 2, \quad x \geqslant 0.$

You should indicate the region for which the inequalities hold by labelling the region R. **[3]**

9. Nov/2021/Paper_H240/03/No.4

   (a) Sketch, on a single diagram, the following graphs.

      • $y = |x - 1|$

      • $y = \dfrac{k}{x}$, where k is a negative constant   **[2]**

   (b) Hence explain why the equation $x|x - 1| = k$ has exactly one real root for any negative value of k.   **[1]**

   (c) Determine the real root of the equation $x|x - 1| = -6$.   **[2]**

10. Nov/2020/Paper_H240/01/No.2

   Simplify fully.
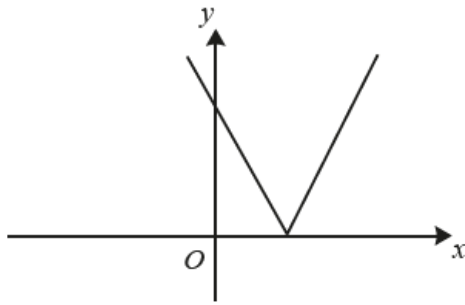
   (a) $\sqrt{12a} \times \sqrt{3a^5}$ **[2]**

   (b) $(64b^3)^{\frac{1}{3}} \times (4b^4)^{-\frac{1}{2}}$ **[2]**

   (c) $7 \times 9^{3c} - 4 \times 27^{2c}$ **[4]**

11. Nov/2020/Paper_H240/01/No.9

The diagram shows the graph of $y = |2x - 3|$.

(a) State the coordinates of the points of intersection with the axes. **[2]**

(b) Given that the graphs of $y = |2x - 3|$ and $y = ax + 2$ have two distinct points of intersection, determine

    (i) the set of possible values of a, **[4]**

    (ii) the x-coordinates of the points of intersection of these graphs, giving your answers in terms of a. **[3]**

12. Nov/2020/Paper_H240/02/No.2

Simplify fully $\dfrac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$. **[4]**

13. Nov/2020/Paper_H240/03/No.2

A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. [3]

14. Nov/2020/Paper_H240/03/No.3

The functions f and g are defined for all real values of x by

$f(x) = 2x^2 + 6x$ and $g(x) = 3x + 2$.

(a) Find the range of f. [3]

(b) Give a reason why f has no inverse. [1]

(c) Given that $fg(-2) = g^{-1}(a)$, where a is a constant, determine the value of a. [4]

(d) Determine the set of values of x for which $f(x) > g(x)$. Give your answer in set notation. [3]