Complex Numbers – 2021/20 GCE AS Pure Further Mathematics A

1. Nov/2021/Paper_Y531/01/No.4

(a) A locus C_1 is defined by $C_1 = \{z : |z+\mathrm{i}| \leqslant |z-2|\}$.

(i) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_1. **[2]**

(ii) Find the cartesian equation of the boundary line of the region representing C_1, giving your answer in the form $ax+by+c=0$. **[2]**

(b) A locus C_2 is defined by $C_2 = \{z : |z+1| \leqslant 3\} \cap \{z : |z-2\mathrm{i}| \geqslant 2\}$.

Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_2. **[3]**

2. Nov/2021/Paper_Y531/01/No.6

In this question you must show detailed reasoning.

(a) Solve the equation $2z^2 - 10z + 25 = 0$ giving your answers in the form $a+b\mathrm{i}$. **[2]**

(b) Solve the equation $3\omega - 2 = \mathrm{i}(5+2\omega)$ giving your answer in the form $a+b\mathrm{i}$. **[4]**

3. Nov/2020/Paper_Y531/01/No.1

In this question you must show detailed reasoning.

Use an algebraic method to find the square roots of $-77 - 36\mathrm{i}$. **[6]**

4. Nov/2020/Paper_Y531/01/No.3

In this question you must show detailed reasoning.

The complex number $7 - 4\mathrm{i}$ is denoted by z.

(a) Giving your answers in the form $a + b\mathrm{i}$, where a and b are rational numbers, find the following.

(i) $3z - 4z^*$ **[2]**

(ii) $(z+1-3\mathrm{i})^2$ **[2]**

(iii) $\dfrac{z+1}{z-1}$ **[2]**

(b) Express z in modulus-argument form giving the modulus exactly and the argument correct to 3 significant figures. **[3]**

(c) The complex number ω is such that $z\omega = \sqrt{585}(\cos(0.5) + \mathrm{i}\sin(0.5))$.

Find the following.

- $|\omega|$
- $\arg(\omega)$, giving your answer correct to 3 significant figures **[3]**

5. Nov/2020/Paper_Y531/01/No.8

Two loci, C_1 and C_2, are defined by

$$C_1 = \left\{z : |z| = |z - 4d^2 - 36|\right\}$$

$$C_2 = \left\{z : \arg(z - 12d - 3\mathrm{i}) = \frac{1}{4}\pi\right\}$$

where d is a real number.

(a) Find, in terms of d, the complex number which is represented on an Argand diagram by the point of intersection of C_1 and C_2.

[You may assume that $C_1 \cap C_2 \neq \varnothing$.] **[6]**

(b) Explain why the solution found in part **(a)** is not valid when $d = 3$. **[2]**