Differentiation – 2021/20 GCE AS Mathematics A

1. Oct/2021/Paper_H230/01/No.2

The number of people, n, living in a small town is changing over time. In an attempt to predict the future growth of the town, a researcher uses the following model for n in terms of t, where t is the time in years from the start of the research.

$$n = 12\,500 + \frac{5000}{t}, \text{ for } t \geqslant 1$$

Find the rate of change of n when $t = 5$. **[4]**

2. Oct/2021/Paper_H230/01/No.5

The fuel consumption of a car, C miles per gallon, varies with the speed, v miles per hour. Jamal models the fuel consumption of his car by the formula

$$C = \tfrac{12}{5}v - \tfrac{3}{125}v^2, \text{ for } 0 \leqslant v \leqslant 80.$$

(a) Suggest a reason why Jamal has included an upper limit in his model. **[1]**

(b) Determine the speed that gives the maximum fuel consumption. **[4]**

Amaya's car does more miles per gallon than Jamal's car. She proposes to model the fuel consumption of her car using a formula of the form

$$C = \tfrac{12}{5}v - \tfrac{3}{125}v^2 + k, \text{ for } 0 \leqslant v \leqslant 80, \text{ where } k \text{ is a positive constant.}$$

(c) Give a reason why this model is **not** suitable. **[1]**

(d) Suggest a different change to Jamal's formula which would give a more suitable model. **[2]**

3. Oct/2021/Paper_H230/01/No.9

In this question you must show detailed reasoning.

Find the equation of the straight line with positive gradient that passes through (0, 2) and is a tangent to the curve $y = x^2 - x + 6$. **[6]**

4. Oct/2021/Paper_H230/02/No.4

The quadratic polynomial $2x^2 - 3$ is denoted by $\mathrm{f}(x)$.

Use differentiation from first principles to determine the value of $\mathrm{f}'(2)$. **[5]**

5. Oct/2020/Paper_H230/01/No.1(a)

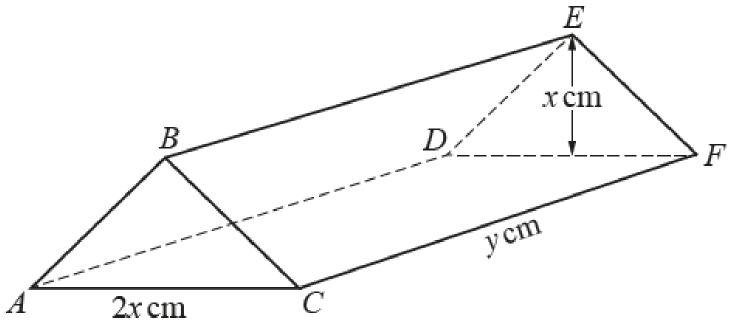
(a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 - 3x + \frac{5}{x^2}\right)$. **[3]**

6. Oct/2020/Paper_H230/02/No.3

In this question you must show detailed reasoning.

Find the equation of the normal to the curve $y = 4\sqrt{x} - 3x + 1$ at the point on the curve where $x = 4$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

7. Oct/2020/Paper_H230/02/No.7



The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism $ABCDEF$. The base $ACFD$ of the roof is a horizontal rectangle, and the cross-section ABC of the roof is an isosceles triangle with $AB = BC$.

The lengths of AC and CF are $2x$ cm and y cm respectively, and the height of BE above the base of the roof is x cm.

The total surface area of the **five** faces of the roof is 600 cm^2 and the volume of the roof is $V\text{ cm}^3$.

(a) Show that $V = kx(300 - x^2)$, where $k = \sqrt{a} + b$ and a and b are integers to be determined. **[6]**

(b) Use differentiation to determine the value of x for which the volume of the roof is a maximum. **[4]**

(c) Find the maximum volume of the roof. Give your answer in cm^3, correct to the nearest integer. **[1]**

(d) Explain why, for this roof, x must be less than a certain value, which you should state. **[2]**

8. June/2019/Paper_H230/01/No.1(a_ b)

It is given that $\mathrm{f}(x) = 3x - \dfrac{5}{x^3}$.

Find

(a) $\mathrm{f}'(x)$, **[3]**

(b) $\mathrm{f}''(x)$, **[2]**

9. June/2019/Paper_H230/01/No.4

(a) Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x$. **[4]**

(b) The equation $x^3 - 6x^2 + 9x + k = 0$ has exactly one real root.

Using your answers from part **(a)** or otherwise, find the range of possible values of k. **[2]**

10. June/2019/Paper_H230/01/No.7

(a) Write down an expression for the gradient of the curve $y = e^{kx}$. **[1]**

(b) The line L is a tangent to the curve $y = e^{\frac{1}{2}x}$ at the point where $x = 2$.

Show that L passes through the point (0, 0). **[4]**

(c) Find the coordinates of the point of intersection of the curves $y = 3e^x$ and $y = 1 - 2e^{\frac{1}{2}x}$. **[6]**

11. June/2019/Paper_H230/02/No.2

(a) Express $5x^2 - 20x + 3$ in the form $p(x+q)^2 + r$, where p, q and r are integers. **[3]**

(b) State the coordinates of the minimum point of the curve $y = 5x^2 - 20x + 3$. **[2]**

(c) State the equation of the normal to the curve $y = 5x^2 - 20x + 3$ at its minimum point. **[1]**