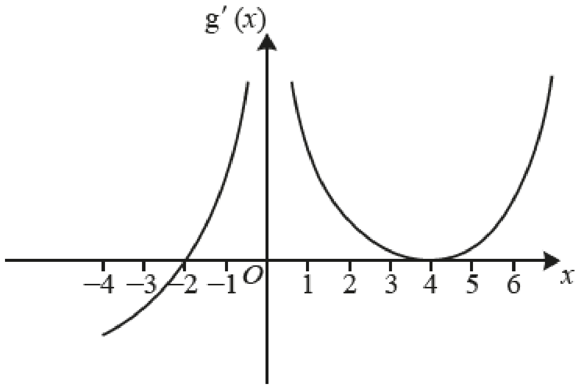
Differentiation – 2021/20 GCE Pure Mathematics A

1. Nov/2021/Paper_H240/01/No.5(b)

(b)



The diagram shows part of the graph of $y = g'(x)$. This is the graph of the gradient function of $y = g(x)$. The graph intersects the x-axis at $x = -2$ and $x = 4$.

(i) State the x-coordinate of any stationary points on the graph of $y = g(x)$. **[1]**

(ii) State the set of values of x for which $y = g(x)$ is a decreasing function. **[1]**

(iii) State the x-coordinate of any points of inflection on the graph of $y = g(x)$. **[1]**

2. Nov/2021/Paper_H240/01/No.9

A particle moves in the x-y plane so that at time t seconds, where $t \geqslant 0$, its coordinates are given by

$x = e^{2t} - 4e^{t} + 3,\ y = 2e^{-3t}$.

(a) Explain why the path of the particle never crosses the x-axis. **[1]**

(b) Determine the exact values of t when the path of the particle intersects the y-axis. **[2]**

(c) Show that $\dfrac{dy}{dx} = \dfrac{3}{2e^{4t} - e^{5t}}$. **[4]**

(d) Hence find the coordinates of the particle when its path is parallel to the y-axis. **[3]**

3. Nov/2021/Paper_H240/01/No.12

A cake is cooling so that, t minutes after it is removed from an oven, its temperature is θ °C. When the cake is removed from the oven, its temperature is 160 °C. After 10 minutes its temperature has fallen to 125 °C.

(a) In a simple model, the rate of decrease of the temperature of the cake is assumed to be constant.

(i) Write down a differential equation for this model. **[1]**

(ii) Solve this differential equation to find θ in terms of t. **[2]**

(iii) State **one** limitation of this model. **[1]**

4. Nov/2021/Paper_H240/02/No.1

Differentiate the following with respect to x.

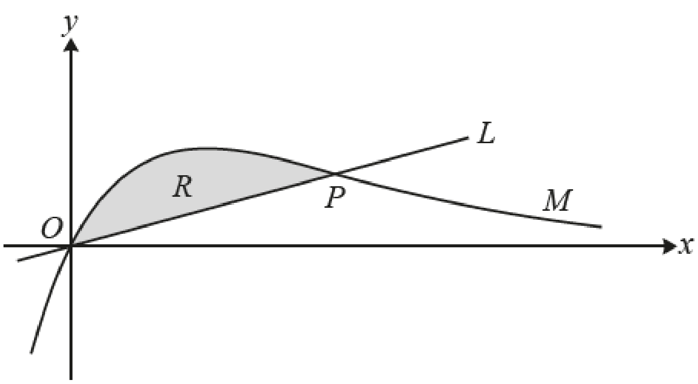
(a) e^{-4x} **[2]**

(b) $\frac{x^2}{x+1}$ **[2]**

5. Nov/2021/Paper_H240/02/No.7

Differentiate $\cos x$ with respect to x, from first principles. **[4]**

6. Nov/2021/Paper_H240/03/No.8



The diagram shows the curve M with equation $y = xe^{-2x}$.

(a) Show that M has a point of inflection at the point P where $x = 1$. **[5]**

The line L passes through the origin O and the point P. The shaded region R is enclosed by the curve M and the line L.

(b) Show that the area of R is given by

$\frac{1}{4}(a + be^{-2})$,

where a and b are integers to be determined. **[6]**

7. Nov/2020/Paper_H240/01/No.3

A cylindrical metal tin of radius r cm is closed at both ends. It has a volume of $16000\pi\,\text{cm}^3$.

(a) Show that its total surface area, $A\,\text{cm}^2$, is given by $A = 2\pi r^2 + 32000\pi r^{-1}$. **[4]**

(b) Use calculus to determine the minimum total surface area of the tin. You should justify that it is a minimum. **[6]**

8. Nov/2020/Paper_H240/01/No.8

(a) Differentiate $(2+3x^2)e^{2x}$ with respect to x. **[3]**

(b) Hence show that $(2+3x^2)e^{2x}$ is increasing for all values of x. **[4]**

9. Nov/2020/Paper_H240/01/No.12

Find the general solution of the differential equation

$$(2x^3 - 3x^2 - 11x + 6)\frac{dy}{dx} = y(20x - 35).$$

Give your answer in the form $y = f(x)$. **[9]**

10. Nov/2020/Paper_H240/02/No.1(a)

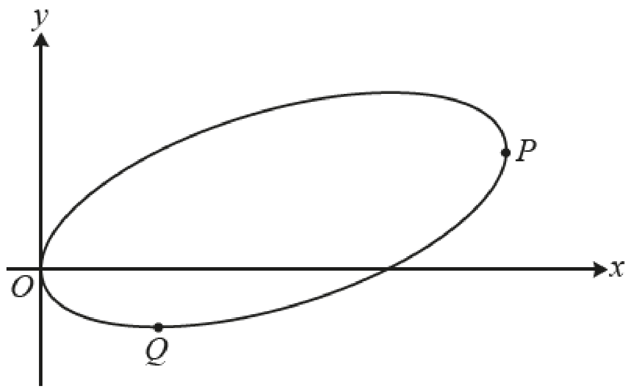
(a) Differentiate the following with respect to x.

(i) $(2x+3)^7$ **[2]**

(ii) $x^3 \ln x$ **[3]**

11. Nov/2020/Paper_H240/03/No.6

In this question you must show detailed reasoning.



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**