Dimensional Analysis – 2021/20 GCE AS Mechanics Further Mathematics A

1. Nov/2021/Paper_Y533/01/No.5

The escape speed of an **unpowered** object is the minimum speed at which it must be projected to escape the gravitational influence of the Earth if it is projected vertically upwards from the Earth's surface. A formula for the escape speed U of an unpowered object of mass m is $U = \sqrt{\frac{2Gm}{r}}$ where r is the radius of the Earth and G is a constant.

(a) Show that the dimensions of G are $\mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$. **[3]**

A rocket is a **powered** object. A rocket is launched with a given **launch speed** and is then powered by engines which apply a constant force for a period of time after the launch.

A student wishes to apply the formula given above to a rocket launch. They wish to model the minimum launch speed required for a rocket to escape the Earth's gravitational influence.

They realise that the given formula is for unpowered objects and so they include an extra term in the formula to obtain $V = \sqrt{\frac{2Gm}{r}} - kP^{\alpha}W^{\beta}t^{\gamma}$.

In their modified formula, G and r are the same as before. The other variables are defined as follows.

- V is the required minimum launch speed of the rocket
- k, α, β and γ are dimensionless constants
- P is the power developed by the engines of the rocket
- m is the initial mass of the rocket
- W is the initial weight of the rocket
- t is the total time for which the engines of the rocket operate

(b) Use dimensional analysis to determine the values of α, β and γ. **[6]**

(c) By considering the value of γ found in part **(b)** explain the relationship between t and V. **[1]**

2. Nov/2020/Paper_Y533/01/No.5

A particle of mass m moves in a straight line with constant acceleration a. Its initial and final velocities are u and v respectively and its final displacement from its starting position is s. In order to model the motion of the particle it is suggested that the velocity is given by the equation

$$v^2 = pu^{\alpha} + qa^{\beta}s^{\gamma}$$

where p and q are dimensionless constants.

(a) Explain why α must equal 2 for the equation to be dimensionally consistent. **[2]**

(b) By using dimensional analysis, determine the values of β and γ. **[4]**

(c) By considering the case where $s = 0$, determine the value of p. **[1]**

(d) By multiplying both sides of the equation by $\frac{1}{2}m$, and using the numerical values of α, β and γ, determine the value of q. **[2]**