Discrete Random Variables – 2021/20 GCE AS Statistics Further Mathematics A

1. Nov/2021/Paper_Y532/01/No.1

The discrete random variable A has the following probability distribution.

a	1	2	5	10	20
$P(A = a)$	0.3	0.1	0.1	0.2	0.3

(a) Find the value of $E(A)$. **[2]**

(b) Determine the value of $Var(A)$. **[3]**

(c) The variable A represents the value in pence of a coin chosen at random from a pile. Mia picks one coin at random from the pile. She then adds, from a different source, another coin of the same value as the one that she has chosen, and one 50p coin.

(i) Find the mean of the value of the three coins. **[2]**

(ii) Find the variance of the value of the three coins. **[1]**

2. Nov/2021/Paper_Y532/01/No.4

Two random variables X and Y have the distributions $B(m, p)$ and $B(n, p)$ respectively, where $p > 0$. It is known that

- $E(Y) = 2E(X)$
- $Var(Y) = 1.2E(X)$.

Determine the value of p. **[4]**

3. Nov/2021/Paper_Y532/01/No.5

The discrete random variable X has a geometric distribution. It is given that $Var(X) = 20$.

Determine $P(X \geqslant 7)$. **[6]**

4. Nov/2021/Paper_Y532/01/No.8

(a) A substance emits particles randomly at a constant average rate of 3.2 per minute. A second substance emits particles randomly, and independently of the first source, at a constant average rate of 2.7 per minute.

Find the probability that the total number of particles emitted by the two sources in a ten-minute period is less than 70. **[3]**

(b) The random variable X represents the number of particles emitted by a substance in a fixed time interval t minutes. It may be assumed that particles are emitted randomly and independently of each other.

In general, the rate at which particles are emitted is proportional to the mass of the substance, but each particle emitted reduces the mass of the substance.

Explain why a Poisson distribution may **not** be a valid model for X if the value of t is very large. **[1]**

(c) The random variable Y has the distribution $\mathrm{Po}(\lambda)$. It is given that

$$\mathrm{P}(Y = r) = \mathrm{P}(Y = r+1)$$

$$\mathrm{P}(Y = r) = 1.5 \times \mathrm{P}(Y = r-1).$$

Determine the following, in either order.

- The value of r
- The value of λ **[7]**

5. Nov/2020/Paper_Y532/01/No.2

Every time a spinner is spun, the probability that it shows the number 4 is 0.2, independently of all other spins.

(a) A pupil spins the spinner repeatedly until it shows the number 4.

Find the mean of the number of spins required. **[2]**

(b) Calculate the probability that the number of spins required is between 3 and 10 inclusive. **[2]**

(c) Each pupil in a class of 30 spins the spinner until it shows the number 4. Out of the 30 pupils, the number of pupils who require at least 10 spins is denoted by X.

Determine the variance of X. **[4]**

6. Nov/2020/Paper_Y532/01/No.6

A statistician investigates the number, F, of signal failures per week on a railway network.

(a) The statistician assumes that signal failures occur randomly.

Explain what this statement means. **[1]**

(b) State two further assumptions needed for F to be well modelled by a Poisson distribution. **[2]**

In a random sample of 50 weeks, the statistician finds that the mean number of failures per week is 1.61, with standard deviation 1.28.

(c) Explain whether this suggests that F is likely to be well modelled by a Poisson distribution. **[2]**

Assume first that $F \sim \text{Po}(1.61)$.

(d) Write down an exact expression for $\text{P}(F = 0)$. **[1]**

(e) Complete the table in the Printed Answer Booklet to show the probabilities of different values of F, correct to three significant figures. **[2]**

Value of F	0	1	$\geqslant 2$
Probability	0.200		

After further investigation, the statistician decides to use a different model for the distribution of F. In this model it is now assumed that $\text{P}(F = 0)$ is still 0.200, but that if one failure occurs, there is an increased probability that further failures occur.

(f) Explain the effect of this assumption on the value of $\text{P}(F = 1)$. **[2]**