Further Vectors – 2021/20 GCE AS Pure Further Mathematics A

1. Nov/2021/Paper_Y531/01/No.1

The lines l_1 and l_2 have the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ -11 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -6 \\ 11 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

(a) Show that l_1 and l_2 intersect. **[4]**

(b) Write down the point of intersection of l_1 and l_2. **[1]**

2. Nov/2021/Paper_Y531/01/No.9

The points $P(3, 5, -21)$ and $Q(-1, 3, -16)$ are on the ceiling of a long straight underground tunnel. A ventilation shaft must be dug from the point M on the ceiling of the tunnel midway between P and Q to horizontal ground level (where the z-coordinate is 0). The ventilation shaft must be perpendicular to the tunnel.

The path of the ventilation shaft is modelled by the vector equation $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ is the position vector of M.

You are given that $\mathbf{b} = \begin{pmatrix} 1 \\ s \\ t \end{pmatrix}$ where s and t are real numbers.

(a) Show that $s = 2.5t - 2$. **[3]**

(b) Show that at the point where the ventilation shaft reaches the ground $\lambda = \frac{c}{t}$, where c is a constant to be determined. **[3]**

(c) Using the results in parts **(a)** and **(b)**, determine the shortest possible length of the ventilation shaft. **[6]**

(d) Explain what the fact that $\mathbf{b} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \neq \mathbf{0}$ means about the direction of the ventilation shaft. **[1]**

3. Nov/2020/Paper_Y531/01/No.7

The equations of two **intersecting** lines are

$$\mathbf{r} = \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

where a is a constant.

(a) Find a vector, **b**, which is perpendicular to both lines. **[2]**

(b) Show that $\mathbf{b}.\begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} = \mathbf{b}.\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$. **[2]**

(c) Hence, or otherwise, find the value of a. **[2]**