Groups – 2021/20 GCE AS Additional Pure Further Mathematics A

1. Nov/2021/Paper_Y535/01/No.6

The set S consists of the following four complex numbers.

$$\sqrt{3}+\mathrm{i} \quad -\sqrt{3}-\mathrm{i} \quad 1-\mathrm{i}\sqrt{3} \quad -1+\mathrm{i}\sqrt{3}$$

For $z_1, z_2 \in S$, the binary operation $\bigcirc$ is defined by $z_1 \bigcirc z_2 = \frac{1}{4}(1+\mathrm{i}\sqrt{3})z_1 z_2$.

(a) (i) Complete the Cayley table for $(S, \bigcirc)$ given in the Printed Answer Booklet. **[3]**

(ii) Verify that $(S, \bigcirc)$ is a group. **[4]**

(iii) State the order of each element of $(S, \bigcirc)$. **[1]**

(b) Write down the only proper subgroup of $(S, \bigcirc)$. **[1]**

(c) (i) Explain why $(S, \bigcirc)$ is a cyclic group. **[1]**

(ii) List all possible generators of $(S, \bigcirc)$. **[1]**

2. Nov/2020/Paper_Y535/01/No.4

(a) For the set $S = \{2, 4, 6, 8, 10, 12\}$, under the operation $\times_{14}$ of multiplication modulo 14, complete the Cayley table given in the Printed Answer Booklet. **[4]**

(b) Show that $(S, \times_{14})$ forms a group, G. (You may assume that $\times_{14}$ is associative.) **[4]**

(c) (i) Write down all the proper subgroups of G. **[2]**

(ii) Given that G is cyclic, write down all the possible generators of G. **[2]**