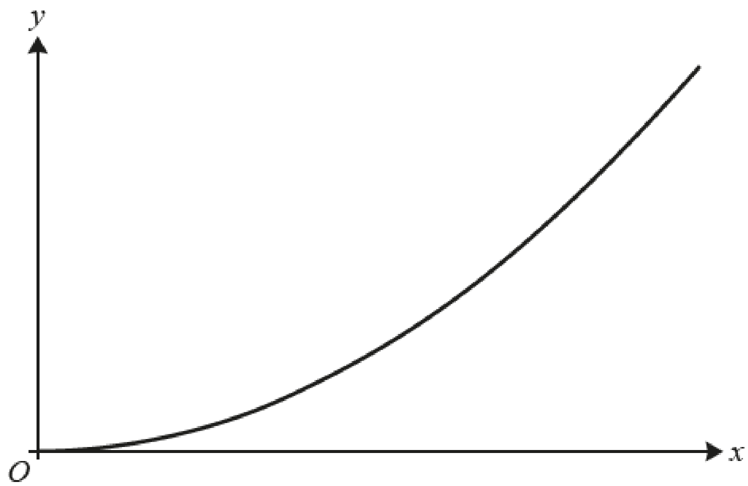
Integration – 2021/20 GCE Pure Mathematics A

1. Nov/2021/Paper_H240/01/No.11

(a) Use the substitution $u^2 = x^2 + 3$ to show that $\int \frac{4x^3}{\sqrt{x^2+3}}\,dx = \frac{4}{3}(x^2 - 6)\sqrt{x^2+3} + c$. **[5]**

(b) **In this question you must show detailed reasoning.**



The graph shows part of the curve $y = \frac{4x^3}{\sqrt{x^2+2}}$.

Find the exact area enclosed by the curve $y = \frac{4x^3}{\sqrt{x^2+3}}$, the normal to this curve at the point (1, 2) and the x-axis. **[7]**

2. Nov/2021/Paper_H240/03/No.7

A curve C in the x-y plane has the property that the gradient of the tangent at the point $P(x, y)$ is three times the gradient of the line joining the point (3, 2) to P.

(a) Express this property in the form of a differential equation. **[2]**

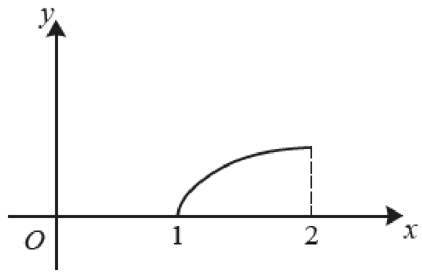
It is given that C passes through the point (4, 3) and that $x > 3$ and $y > 2$ at all points on C.

(b) Determine the equation of C giving your answer in the form $y = f(x)$. **[4]**

The curve C may be obtained by a transformation of part of the curve $y = x^3$.

(c) Describe fully this transformation. **[2]**

3. Nov/2020/Paper_H240/01/No.10(b)



The diagram shows the curve $y = \sin\left(\frac{1}{2}\sqrt{x-1}\right)$, for $1 \leqslant x \leqslant 2$.

(b) **(i)** Use the substitution $t = \sqrt{x-1}$ to show that $\int \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx = \int 2t\sin\left(\frac{1}{2}t\right)dt$. **[3]**

(ii) Hence show that $\int_1^2 \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx = 8\sin\frac{1}{2} - 4\cos\frac{1}{2}$. **[4]**

4. Nov/2020/Paper_H240/02/No.1(b, c)

(b) Find $\int \cos 5x\,dx$. **[2]**

(c) Find the equation of the curve through (1, 3) for which $\frac{dy}{dx} = 6x - 5$. **[2]**

5. Nov/2020/Paper_H240/02/No.8

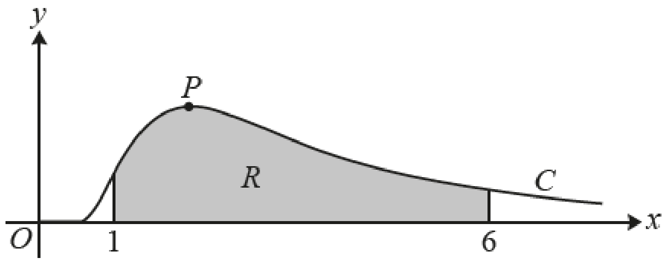
The rate of change of a certain population P at time t is modelled by the equation $\frac{dP}{dt} = (100 - P)$.

Initially $P = 2000$.

(a) Determine an expression for P in terms of t. **[7]**

(b) Describe how the population changes over time. **[2]**

6. Nov/2020/Paper_H240/03/No.5



The diagram shows the curve C with parametric equations

$x = \frac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. **[4]**

The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

(b) Show that the area of R is given by

$$\int_a^b 3te^{-2t}\,dt,$$

where a and b are constants to be determined. **[3]**

(c) Hence determine the exact area of R. **[5]**