Matrices – 2021/20 GCE AS Pure Further Mathematics A

1. Nov/2021/Paper_Y531/01/No.5

Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$.

(a) Use **A** and **B** to disprove the proposition: "Matrix multiplication is commutative". **[2]**

Matrix **B** represents the transformation T_B.

(b) Describe the transformation T_B. **[2]**

(c) By considering the inverse transformation of T_B, determine $\mathbf{B}^{-1}$. **[2]**

Matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$ and represents the transformation T_C.

The transformation T_{BC} is transformation T_C followed by transformation T_B.

An object shape of area 5 is transformed by T_{BC} to an image shape *N*.

(d) Determine the area of *N*. **[2]**

2. Nov/2021/Paper_Y531/01/No.8

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} t-1 & t-1 & t-1 \\ 1-t & 6 & t \\ 2-2t & 2-2t & 1 \end{pmatrix}$.

(a) Find, in fully factorised form, an expression for det **A** in terms of *t*. **[3]**

(b) State the values of *t* for which **A** is singular. **[1]**

You are given the following system of equations in *x*, *y* and *z*, where *b* is a real number.

$$\begin{aligned} (b^2+1)x + (b^2+1)y + (b^2+1)z &= 5 \\ (-b^2-1)x + 6y + (b^2+2)z &= 10 \\ (-2b^2-2)x + (-2b^2-2)y + z &= 15 \end{aligned}$$

(c) Determine which **one** of the following statements about the solution of the equations is true.

- There is a unique solution for all values of *b*.
- There is a unique solution for some, but not all, values of *b*.
- There is no unique solution for any value of *b*. **[2]**

3. Nov/2020/Paper_Y531/01/No.2

P, Q and T are three transformations in 2-D.

P is a reflection in the x-axis. **A** is the matrix that represents P.

(a) Write down the matrix **A**. **[1]**

Q is a shear in which the y-axis is invariant and the point $\begin{pmatrix}1\\0\end{pmatrix}$ is transformed to the point $\begin{pmatrix}1\\2\end{pmatrix}$. **B** is the matrix that represents Q.

(b) Find the matrix **B**. **[2]**

T is P followed by Q. **C** is the matrix that represents T.

(c) Determine the matrix **C**. **[2]**

L is the line whose equation is $y = x$.

(d) Explain whether or not L is a line of invariant points under T. **[2]**

An object parallelogram, M, is transformed under T to an image parallelogram, N.

(e) Explain what the value of the determinant of **C** means about

- the area of N compared to the area of M,
- the orientation of N compared to the orientation of M. **[3]**

4. Nov/2020/Paper_Y531/01/No.4

You are given the system of equations

$$a^2x - 2y = 1$$
$$x + b^2y = 3$$

where a and b are real numbers.

(a) Use a matrix method to find x and y in terms of a and b. **[4]**

(b) Explain why the method used in part **(a)** works for all values of a and b. **[2]**