Motion in a circle – 2021/20 GCE AS Mechanics Further Mathematics A

1. Nov/2021/Paper_Y533/01/No.1

One end of a light inextensible string of length 2.8 m is attached to a fixed point O on a smooth horizontal table. The other end of the string is attached to a particle P which moves on the table, with the string taut, in a circular path around O. The speed of P is constant and P completes each circle in 0.84 seconds.

(a) Find the magnitude of the angular velocity of P. **[2]**

(b) Find the speed of P. **[2]**

(c) Find the magnitude of the acceleration of P. **[2]**

(d) State the direction of the acceleration of P. **[1]**

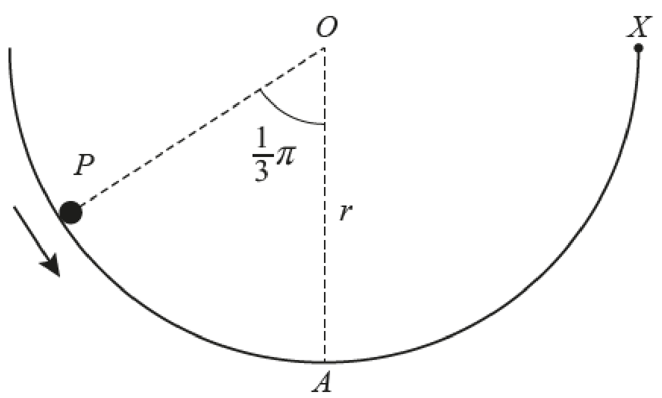
2. Nov/2021/Paper_Y533/01/No.6

A smooth hemispherical shell of radius r m is held with its circular rim horizontal and uppermost. The centre of the rim is at the point O and the point on the inner surface directly below O is A.

A small object P of mass m kg is held at rest on the inner surface of the shell so that $\angle POA = \frac{1}{3}\pi$ radians. At the instant that P is released, an impulse is applied to P in the direction of the tangent to the surface at P in the vertical plane containing O, A and P. The magnitude of the impulse is denoted by I Ns.

P immediately starts to move along the surface towards A (see diagram).

X is a point on the circular rim. P leaves the shell at X.



In an initial model of the motion of P it is assumed that P experiences no resistance to its motion.

(a) Find in terms of r, g, m and I an expression for the speed of P at the instant that it leaves the shell at X. **[4]**

(b) Find in terms of r, g, m and I an expression for the maximum height attained by P above X after it has left the shell. **[1]**

(c) Find an expression for the maximum mass of P for which P still leaves the shell. **[2]**

In a revised model it is assumed that P experiences a resistive force of constant magnitude R while it is moving.

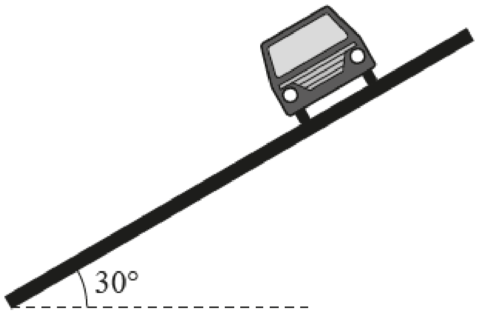
(d) Show that, in order for P to **still** leave the shell at X under the revised model,

$$I > \sqrt{m^2gr + \frac{5\pi mrR}{3}}.$$ **[3]**

(e) Show that the inequality from part **(d)** is dimensionally consistent. **[2]**

3. Nov/2020/Paper_Y533/01/No.7

It is required to model the motion of a car of mass m kg travelling at a constant speed v m s^{-1} around a circular portion of banked track. The track is banked at 30° (see diagram).



In a model, the following modelling assumptions are made.

- The track is smooth.
- The car is a particle.
- The car follows a horizontal circular path with radius r m.

(a) Show that, according to the model, $\sqrt{3}v^2 = gr$. **[4]**

For a particular portion of banked track, $r = 24$.

(b) Find the value of v as predicted by the model. **[2]**

A car is being driven on this portion of the track at the constant speed calculated in part **(b)**. The driver finds that in fact he can drive a little slower or a little faster than this while still moving in the same horizontal circle.

(c) Explain

- how this contrasts with what the model predicts,
- how to improve the model to account for this. **[3]**