Number Theory – 2021/20 GCE AS Additional Pure Further Mathematics A

1. Nov/2021/Paper_Y535/01/No.4

(a) Let $a = 1071$ and $b = 67$.

(i) Find the unique integers q and r such that $a = bq + r$, where $q > 0$ and $0 \leqslant r < b$. **[1]**

(ii) Hence express the answer to **(a)(i)** in the form of a linear congruence modulo b. **[1]**

(b) Use the fact that $358 \times 715 - 239 \times 1071 = 1$ to prove that 715 and 1071 are co-prime. **[4]**

2. Nov/2021/Paper_Y535/01/No.7

(a) Let $\mathrm{f}(n) = 2^{4n+3} + 3^{3n+1}$.

Use arithmetic modulo 11 to prove that $\mathrm{f}(n) \equiv 0 \pmod{11}$ for all integers $n \geqslant 0$. **[4]**

(b) Use the standard test for divisibility by 11 to prove the following statements.

(i) $10^{33} + 1$ is divisible by 11 **[2]**

(ii) $10^{33} + 1$ is divisible by 121 **[4]**

3. Nov/2020/Paper_Y535/01/No.1

(a) Evaluate 13×19 modulo 31. **[1]**

(b) Solve the linear congruence $13x \equiv 9 \pmod{31}$. **[3]**

4. Nov/2020/Paper_Y535/01/No.3

In this question, N is the number 26 132 652.

(a) Without dividing N by 13, explain why 13 is a factor of N. **[1]**

(b) Use standard divisibility tests to show that 36 is a factor of N. **[3]**

It is given that $N = 36 \times 725\,907$.

(c) Use the results of parts **(a)** and **(b)** to deduce that 13 is a factor of 725 907. **[2]**