Numerical methods – 2021/20 GCE Pure Mathematics A

1. Nov/2021/Paper_H240/01/No.7

The curve $y = (x^2 - 2)\ln x$ has one stationary point which is close to $x = 1$.

(a) Show that the x-coordinate of this stationary point satisfies the equation $2x^2 \ln x + x^2 - 2 = 0$. **[2]**

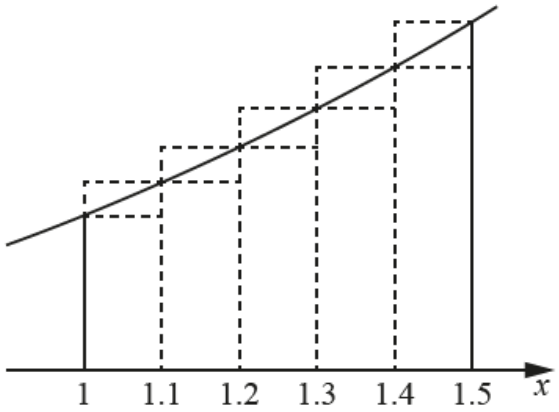
(b) Show that the Newton-Raphson iterative formula for finding the root of the equation in part **(a)** can be written in the form $x_{n+1} = \dfrac{2x_n^2 \ln x_n + 3x_n^2 + 2}{4x_n(\ln x_n + 1)}$. **[4]**

(c) Apply the Newton-Raphson formula with initial value $x_1 = 1$ to find x_2 and x_3. **[1]**

(d) Find the coordinates of this stationary point, giving each coordinate correct to **3** decimal places. **[2]**

2. Nov/2021/Paper_H240/02/No.6

Alex is investigating the area, A, under the graph of $y = x^2$ between $x = 1$ and $x = 1.5$. They draw the graph, together with rectangles of width $\delta x = 0.1$, and varying heights y.



(a) Use the rectangles in the diagram to show that lower and upper bounds for the area A are 0.73 and 0.855 respectively. **[1]**

(b) Alex finds lower and upper bounds for the area A, using widths δx of decreasing size. The results are shown in the table. Where relevant, values are given correct to 3 significant figures.

Width δx	0.1	0.05	0.025	0.0125
Lower bound for area A	0.73	0.761	0.776	0.784
Upper bound for area A	0.855	0.823	0.807	0.799

Use Alex's results to estimate the value of A correct to **2** significant figures. Give a brief justification for your estimate. **[2]**

(c) Write down an expression, in terms of y and δx, for the exact value of the area A. **[2]**

3. Nov/2021/Paper_H240/03/No.6

The equation $6\arcsin(2x-1)-x^2=0$ has exactly one real root.

(a) Show by calculation that the root lies between 0.5 and 0.6. **[2]**

In order to find the root, the iterative formula

$$x_{n+1}=p+q\sin(rx_n^2),$$

with initial value $x_0=0.5$, is to be used.

(b) Determine the values of the constants p, q and r. **[2]**

(c) Hence find the root correct to **4** significant figures. Show the result of each step of the iteration process. **[2]**

4. Nov/2020/Paper_H240/01/No.7

Two students, Anna and Ben, are starting a revision programme. They will both revise for 30 minutes on Day 1. Anna will increase her revision time by 15 minutes for every subsequent day. Ben will increase his revision time by 10% for every subsequent day.

(a) Verify that on Day 10 Anna does 94 minutes more revision than Ben, correct to the nearest minute. **[3]**

Let Day X be the first day on which Ben does more revision than Anna.

(b) Show that X satisfies the inequality $X>\log_{1.1}(0.5X+0.5)+1$. **[3]**

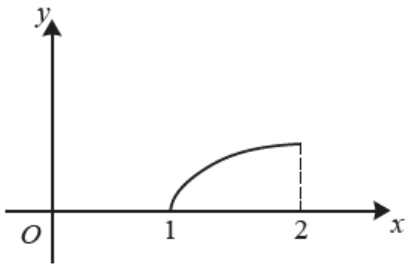
(c) Use the iterative formula $x_{n+1}=\log_{1.1}(0.5x_n+0.5)+1$ with $x_1=10$ to find the value of X.

You should show the result of each iteration. **[3]**

(d) (i) Give a reason why Anna's revision programme may not be realistic. **[1]**

(ii) Give a **different** reason why Ben's revision programme may not be realistic. **[1]**

5. Nov/2020/Paper_H240/01/No.10(a)



The diagram shows the curve $y = \sin\left(\frac{1}{2}\sqrt{x-1}\right)$, for $1 \leqslant x \leqslant 2$.

(a) Use rectangles of width 0.25 to find upper and lower bounds for $\int_1^2 \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx$. Give your answers correct to 3 significant figures. **[4]**

6. Nov/2020/Paper_H240/03/No.4

A curve has equation $y = 2\ln(k-3x) + x^2 - 3x$, where k is a positive constant.

(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. **[5]**

It is also given that the curve intersects the x-axis at exactly one point.

(b) Show by calculation that the x-coordinate of this point lies between 0.5 and 1.5. **[2]**

(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x-coordinate of the point where the curve intersects the x-axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. **[3]**

(d) By choosing suitable bounds, verify that your answer to part **(c)** is correct to 5 decimal places. **[1]**