Proof – 2021/20 GCE Pure Mathematics A

1. Nov/2021/Paper_H240/02/No.8

The number K is defined by $K = n^3 + 1$, where n is an integer greater than 2.

(a) Given that $n^3 + 1 \equiv (n+1)(n^2 + bn + c)$, find the constants b and c. **[1]**

(b) Prove that K has at least two **distinct** factors other than 1 and K. **[5]**

2. Nov/2020/Paper_H240/01/No.4

Prove by contradiction that there is no greatest multiple of 5. **[3]**

3. Nov/2020/Paper_H240/02/No.5

(a) Determine the set of values of n for which $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are positive integers. **[3]**

A 'Pythagorean triple' is a set of three positive integers a, b and c such that $a^2 + b^2 = c^2$.

(b) Prove that, for the set of values of n found in part **(a)**, the numbers n, $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ form a Pythagorean triple. **[2]**