Sequences and Series – 2021/20 GCE AS Additional Pure Further Mathematics A

1. Nov/2021/Paper_Y535/01/No.3

For positive integers n, the sequence of Fibonacci numbers, $\{F_n\}$, starts with the terms $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, … and is given by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ $(n \geqslant 3)$.

(a) Show that $F_{3k+3} = 2F_{3k+1} + F_{3k}$, where k is a positive integer. **[2]**

(b) Prove by induction that F_{3n} is even for all positive integers n. **[4]**

2. Nov/2021/Paper_Y535/01/No.8

A sequence $\{u_n\}$ is defined by the recurrence system

$u_1 = 1$ and $u_{n+1} = a - \frac{a^2}{2u_n}$ for $n \geqslant 1$, where a is a positive constant.

Determine with justification the behaviour of the sequence for all possible values of a. **[7]**

3. Nov/2020/Paper_Y535/01/No.5

(a) Determine the general solution of the first-order recurrence relation $V_{n+1} = 2V_n + n$. **[6]**

(b) Given that $V_1 = 8$, find the exact value of V_{20}. **[2]**

4. Nov/2020/Paper_Y535/01/No.7

In a conservation project, a batch of 100 000 tadpoles which have just hatched from eggs is introduced into an environment which has no frog population. Previous research suggests that for every 1 million tadpoles hatched only 3550 will live to maturity at 12 weeks, when they become adult frogs.

It is assumed that the steady decline in the population of tadpoles, from all causes, can be explained by a weekly death-rate factor, r, which is constant across each week of this twelve-week period.

Let T_k denote the total number of tadpoles alive at the end of k weeks after the start of this project.

(a) (i) Explain why a recurrence system for T_k is given by $T_0 = 100\,000$ and $T_{k+1} = (1-r)T_k$ for $0 \leqslant k \leqslant 12$. **[3]**

(ii) Show that $r = 0.375$, correct to 3 significant figures. **[2]**

The proportion of females within each batch of tadpoles is p, where $0 < p < 1$. In a simple model of the frog population the following assumptions are made.

- The death rate factor for adult frogs is also r and is the same for males and females.
- The frog population will survive provided there are at least **thirty** female frogs alive sixteen weeks after the start of this project.

(b) (i) Find the smallest value of p for which the frog population will survive according to the model. **[3]**

(ii) Write down one assumption that has been made in order to obtain this result. **[1]**

Each surviving female will then lay a batch of eggs from which 2500 tadpoles are hatched.

(c) By considering the total number of tadpoles hatched, give one criticism of the assumption that the frog population will survive provided there are at least **thirty** female frogs alive sixteen weeks after the start of this project. **[1]**