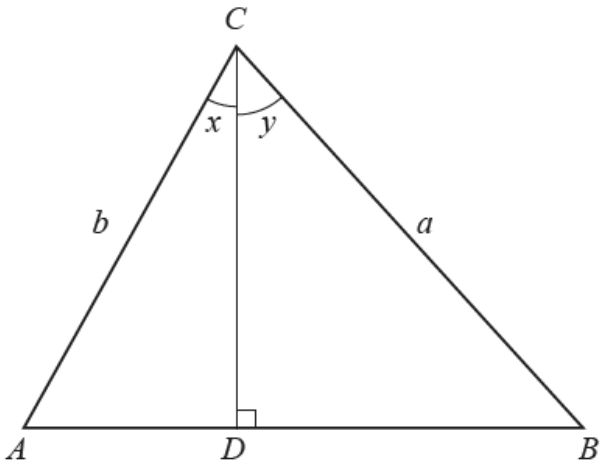
Trigonometry – 2021/20 GCE Pure Mathematics A

1. Nov/2021/Paper_H240/01/No.10

(a)



The diagram shows triangle ABC. The perpendicular from C to AB meets AB at D.

Angle $ACD = x$, angle $DCB = y$, length $BC = a$ and length $AC = b$.

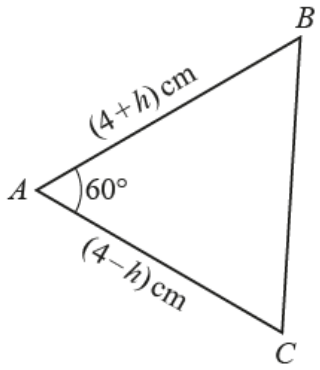
(i) Explain why the length of CD can be written as $a\cos y$. **[1]**

(ii) Show that the area of the triangle ADC is given by $\frac{1}{2}ab\sin x\cos y$. **[1]**

(iii) Hence, or otherwise, show that $\sin(x+y) = \sin x\cos y + \cos x\sin y$. **[4]**

(b) Given that $\sin(30° + \alpha) = \cos(45° - \alpha)$, show that $\tan\alpha = 2 + \sqrt{6} - \sqrt{3} - \sqrt{2}$. **[5]**

2. Nov/2021/Paper_H240/03/No.2(a)



The diagram shows triangle ABC in which angle A is $60°$ and the lengths of AB and AC are $(4+h)$ cm and $(4-h)$ cm respectively.

(a) Show that the length of BC is p cm where

$$p^2 = 16+3h^2.$$ **[2]**

3. Nov/2021/Paper_H240/03/No.5

A particle P moves along a straight line in such a way that at time t seconds P has velocity $v\,\text{m s}^{-1}$, where

$$v = 12\cos t + 5\sin t.$$

(a) Express v in the form $R\cos(t-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to **4** significant figures. **[3]**

(b) Hence find the two smallest positive values of t for which P is moving, in either direction, with a speed of $3\,\text{m s}^{-1}$. **[3]**

4. Nov/2020/Paper_H240/01/No.1

(a) For a small angle θ, where θ is in radians, show that $2\cos\theta + (1-\tan\theta)^2 \approx 3 - 2\theta$. **[3]**

(b) Hence determine an approximate solution to $2\cos\theta + (1-\tan\theta)^2 = 28\sin\theta$. **[2]**

5. Nov/2020/Paper_H240/02/No.4

In this question you must show detailed reasoning.

Solve the equation $3\sin^4\phi + \sin^2\phi = 4$, for $0 \leqslant \phi < 2\pi$, where ϕ is measured in radians. **[5]**

6. Nov/2020/Paper_H240/02/No.6

Prove that $\sqrt{2}\cos(2\theta+45°) \equiv \cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta$, where θ is measured in degrees. **[3]**

7. Nov/2020/Paper_H240/03/No.1

Triangle ABC has $AB = 8.5\,\text{cm}$, $BC = 6.2\,\text{cm}$ and angle $B = 35°$.

Calculate the area of the triangle. **[2]**