Complex Numbers – 2022 GCE Pure Core 1 Further Math A Y540

1. June/2022/Paper_ Y540/01/No.3

In this question you must show detailed reasoning.

(a) Find the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

The loci C_1 and C_2 are given by $|z| = |z - 2i|$ and $|z - 2| = \sqrt{5}$ respectively.

(b) (i) Sketch on a single Argand diagram the loci C_1 and C_2, showing any intercepts with the imaginary axis. **[3]**

(ii) Indicate, by shading on your Argand diagram, the region

$\{z : |z| \leqslant |z - 2i|\} \cap \{z : |z - 2| \leqslant \sqrt{5}\}$. **[1]**

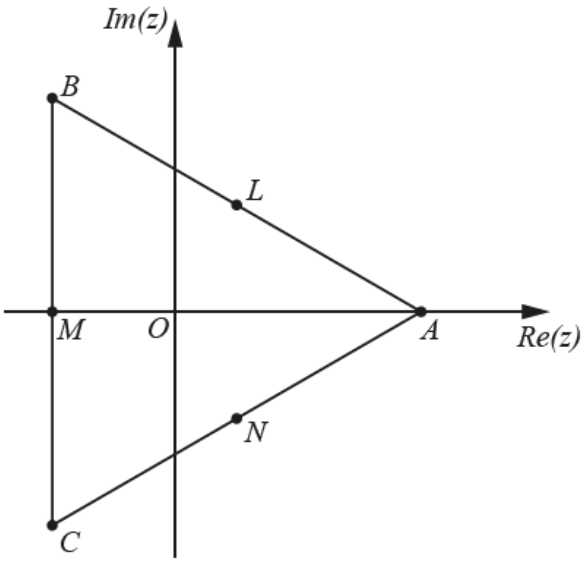
(c) (i) Show that both of the roots of the equation $2z^2 - 2z + 5 = 0$ satisfy $|z - 2| < \sqrt{5}$. **[2]**

(ii) State, with a reason, which root of the equation $2z^2 - 2z + 5 = 0$ satisfies $|z| < |z - 2i|$. **[1]**

(d) On the same Argand diagram as part **(b)**, indicate the positions of the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

2. June/2022/Paper_ Y540/01/No.9

The cube roots of unity are represented on the Argand diagram below by the points A, B and C.



The points L, M and N are the midpoints of the line segments AB, BC and CA respectively.

Determine a degree 6 polynomial equation with integer coefficients whose roots are the complex numbers represented by the points A, B, C, L, M and N. **[5]**