Complex Numbers – 2022 GCE Pure Core 2 Further Math A Y541

1. June/2022/Paper_ Y541/01/No.1

(a) Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. **[1]**

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

(b) Show that the lines intersect, stating the point of intersection. **[5]**

2. June/2022/Paper_ Y541/01/No.9

In this question you must show detailed reasoning.

(a) Show that $\mathrm{Re}\left(e^{4i\theta}(e^{i\theta}+e^{-i\theta})^4\right) = a\cos 4\theta \cos^4\theta$, where a is an integer to be determined. **[3]**

(b) Hence show that $\cos\frac{1}{12}\pi = \frac{1}{2}\sqrt[4]{b+c\sqrt{3}}$, where b and c are integers to be determined. **[6]**