Differential Equations – 2022 GCE Pure Core 1 Further Math A Y540

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A biologist is studying the effect of pesticides on crops. On a certain farm pesticide is regularly applied to a particular crop which grows in soil. Over time, pesticide is transferred between the crop and the soil at a rate which depends on the amount of pesticide in both the crop and the soil. The amount of pesticide in the crop after t days is x grams. The amount of pesticide in the soil after t days is y grams. Initially, when $t = 0$, there is no pesticide in either the crop or the soil.

At first it is assumed that no pesticide is lost from the system. The biologist further assumes that pesticide is added to the crop at a constant rate of k grams per day, where $k > 6$.

After collecting some initial data, the biologist suggests that for $t \geqslant 0$, this situation can be modelled by the following pair of first order linear differential equations.

$$\frac{dx}{dt} = -2x + 78y + k$$
$$\frac{dy}{dt} = 2x - 78y$$

(a) (i) Show that $\frac{d^2x}{dt^2} + 80\frac{dx}{dt} = 78k$. **[2]**

(ii) Determine the particular solution for x in terms of k and t. **[7]**

If more than 250 grams of pesticide is found in the crop, then it will fail food safety standards.

(iii) The crop is tested 50 days after the pesticide is first added to it.

Explain why, according to this model, the crop will fail food safety standards as a result of this test. **[1]**

Further data collection suggests that some pesticide decays in the soil and so is lost from the system. The model is refined in light of this data. The particular solution for x for this refined model is

$$x = k\left(20 - e^{-41t}\left(20\cosh\left(\sqrt{1677}\,t\right) + \frac{819}{\sqrt{1677}}\sinh\left(\sqrt{1677}\,t\right)\right)\right).$$

(b) Given now that $k < 12$, determine whether the crop will fail food safety standards in the long run according to this refined model. **[2]**

In the refined model, it is still assumed that pesticide is added to the crop at a constant rate.

(c) Suggest a reason why it might be more realistic to model the addition of pesticide as not being at a constant rate. **[1]**