Exponentials and Logarithms – 2022 GCE AS Mathematics A

1. June/2022/Paper_H230/01/No.6

During some research the size, P, of a population of insects, at time t months after the start of the research, is modelled by the following formula.

$$P = 100\,e^{t}$$

(a) Use this model to answer the following.

(i) Find the value of P when $t = 4$. **[1]**

(ii) Find the value of t when the population is 9000. **[2]**

(b) It is suspected that a more appropriate model would be the following formula.

$P = ka^{t}$ where k and a are constants.

(i) Show that, using this model, the graph of $\log_{10}P$ against t would be a straight line. **[2]**

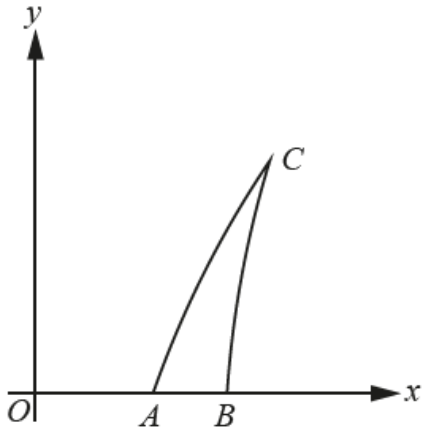
Some observations of t and P gave the following results.

t	1	2	3	4	5
P	100	500	1800	7000	19 000
$\log_{10}P$	2.00	2.70	3.26	3.85	4.28

(ii) On the grid in the Printed Answer Booklet, draw a line of best fit for the data points $(t, \log_{10}P)$ given in the table. **[2]**

(iii) Hence estimate the values of k and a. **[4]**

2. June/2022/Paper_H230/02/No.6



The shape ABC shown in the diagram is a student's design for the sail of a small boat.

The curve AC has equation $y = 2\log_2 x$ and the curve BC has equation $y = \log_2\left(x - \frac{3}{2}\right) + 3$.

(a) State the x-coordinate of point A. **[1]**

(b) Determine the x-coordinate of point B. **[3]**

(c) By solving an equation involving logarithms, show that the x-coordinate of point C is 2. **[4]**

It is given that, correct to 3 significant figures, the area of the sail is 0.656 units^2.

(d) Calculate by how much the area is over-estimated or under-estimated when the curved edges of the sail are modelled as straight lines. **[4]**