

**Further Calculus – 2022 GCE Pure Core 2 Further Math A Y541****1. June/2022/Paper\_ Y541/01/No.6**

A particle,  $P$ , positioned at the origin,  $O$ , is projected with a certain velocity along the  $x$ -axis.  $P$  is then acted on by a single force which varies in such a way that  $P$  moves backwards and forwards along the  $x$ -axis.

When the time after projection is  $t$  seconds, the displacement of  $P$  from the origin is  $x$  m and its velocity is  $v$  ms<sup>-1</sup>.

The motion of  $P$  is modelled using the differential equation  $\ddot{x} + \omega^2 x = 0$ , where  $\omega$  rad s<sup>-1</sup> is a positive constant.

- (a) Write down the general solution of this differential equation. [1]

$D$  is the point where  $x = d$  for some positive constant,  $d$ . When  $P$  reaches  $D$  it comes to instantaneous rest.

- (b) Using the answer to part (a), determine expressions, in terms of  $\omega$ ,  $d$  and  $t$  only, for the following quantities

- $x$
  - $v$
- [3]

- (c) Hence show that, according to the model,  $v^2 = \omega^2(d^2 - x^2)$ . [1]

The quantity  $z$  is defined by  $z = \frac{1}{v}$ .

- (d) Using part (c), determine an expression for  $z_m$ , the mean value of  $z$  with respect to the displacement, as  $P$  moves directly from  $O$  to  $D$ . [2]

One measure of the validity of the model is consideration of the value of  $z_m$ . If  $z_m$  exceeds 8 then the model is considered to be valid.

The value of  $d$  is measured as 0.25 to 2 significant figures. The value of  $\omega$  is measured as  $0.75 \pm 0.02$ .

- (e) Determine what can be inferred about the validity of the model from the given information. [1]

- (f) Find, according to the model, the least possible value of the velocity with which  $P$  was initially projected. Give your answer to 2 significant figures. [2]