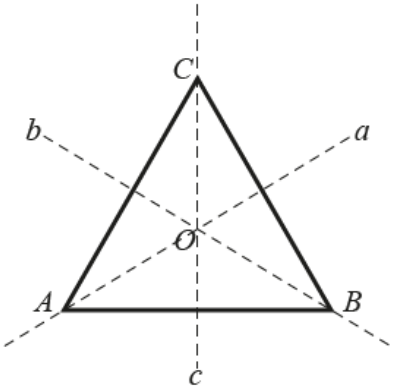
Groups – 2022 GCE AS Additional Pure Further Mathematics A

1. June/2022/Paper_Y535/01/No.7

The diagram below shows an equilateral triangle ABC. The three lines of reflection symmetry of ABC (the lines a, b and c) are shown as broken lines. The point of intersection of these three lines, O, is the centre of rotational symmetry of the triangle.



The group D_3 is defined as the set of symmetries of ABC under the composition of the following transformations.

i : the identity transformation

a : reflection in line a

b : reflection in line b

c : reflection in line c

p : an anticlockwise rotation about O through 120°

q : a clockwise rotation about O through 120°

Note that the lines a, b and c are unaffected by the transformations and remain fixed.

(a) On the diagrams provided in the Printed Answer Booklet, show each of the six elements of D_3 obtained when the above transformations are applied to triangle ABC. **[3]**

(b) Complete the Cayley table given in the Printed Answer Booklet. **[3]**

(c) List all the proper subgroups of D_3. **[3]**

(d) State, with justification, whether D_3 is

(i) cyclic, **[1]**

(ii) abelian. **[1]**

(e) The group H, also of order 6, is the set of rotational symmetries of the regular hexagon. Describe **two structural** differences between D_3 and H. **[2]**