Groups – 2022 GCE Additional Pure Further Math A Y545

1. June/2022/Paper_ Y545/01/No.8

(a) Explain why all groups of even order must contain at least one self-inverse element (that is, an element of order 2). **[2]**

(b) Prove that any group, in which every (non-identity) element is self-inverse, is abelian. **[2]**

(c) A student believes that, if x and y are two distinct, non-identity, self-inverse elements of a group, then the element xy is also self-inverse.

The table shown here is the Cayley table for the non-cyclic group of order 6, having elements i, a, b, c, d and e, where i is the identity.

	i	a	b	c	d	e
i	i	a	b	c	d	e
a	a	i	d	e	b	c
b	b	e	i	d	c	a
c	c	d	e	i	a	b
d	d	c	a	b	e	i
e	e	b	c	a	i	d

By considering the elements of this group, produce a counter-example which proves that this student is wrong. **[2]**

(d) A group G has order $4n+2$, for some positive integer n, and i is the identity element of G. Let x and y be two distinct, non-identity, self-inverse elements of G. By considering the set $H = \{i, x, y, xy\}$, prove by contradiction that not all elements of G are self-inverse. **[4]**