

Integration – 2022 GCE Pure Mathematics A**1. June/2022/Paper_H240/01/No.9**

Use the substitution $x = 2 \sin \theta$ to show that $\int_1^{\sqrt{3}} \sqrt{4-x^2} \, dx = \frac{1}{3}\pi$. [7]

2. June/2022/Paper_H240/01/No.11

The gradient function of a curve is given by $\frac{dy}{dx} = \frac{3x^2 \ln x}{e^{3y}}$.

The curve passes through the point $(e, 1)$.

(a) Find the equation of this curve, giving your answer in the form $e^{3y} = f(x)$. [6]

(b) Show that, when $x = e^2$, the y -coordinate of this curve can be written as $y = a + \frac{1}{3} \ln(b e^3 + c)$, where a , b and c are constants to be determined. [3]

3. June/2022/Paper_H240/02/No.3

(a) Amaya and Ben integrated $(1+x)^2$, with respect to x , using different methods, as follows.

$$\text{Amaya: } \int (1+x)^2 \, dx = \frac{(1+x)^3}{3} + c = \frac{1}{3} + x + x^2 + \frac{1}{3}x^3 + c$$

$$\text{Ben: } \int (1+x)^2 \, dx = \int (1+2x+x^2) \, dx = x + x^2 + \frac{1}{3}x^3 + c$$

Charlie said that, because these answers are different, at least one of them must be wrong.

Explain whether you agree with Charlie's statement. [1]

(b) You are given that a is a constant greater than 1.

(i) Find $\int_1^a \frac{1}{(1+x)^2} \, dx$, giving your answer as a single fraction in terms of the constant a . [3]

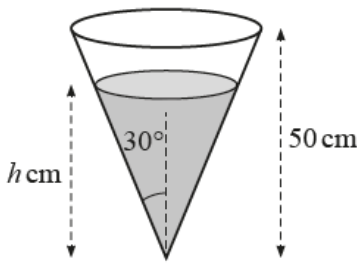
(ii) You are given that the area enclosed by the curve $y = \frac{1}{(1+x)^2}$, the x -axis and the lines $x = 1$ and $x = a$ is equal to $\frac{1}{3}$.

Determine the value of a . [2]

(c) In this question you must show detailed reasoning.

Find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{\cos 2x}{\sin 2x + 2} \, dx$, giving your answer in its simplest form. [4]

4. June/2022/Paper_H240/02/No.8



The diagram shows a water tank which is shaped as an inverted cone with semi-vertical angle 30° and height 50 cm. Initially the tank is full, and the depth of the water is 50 cm.

Water flows out of a small hole at the bottom of the tank. The rate at which the water flows out is modelled by $\frac{dV}{dt} = -2h$, where $V \text{ cm}^3$ is the volume of water remaining and $h \text{ cm}$ is the depth of water in the tank t seconds after the water begins to flow out.

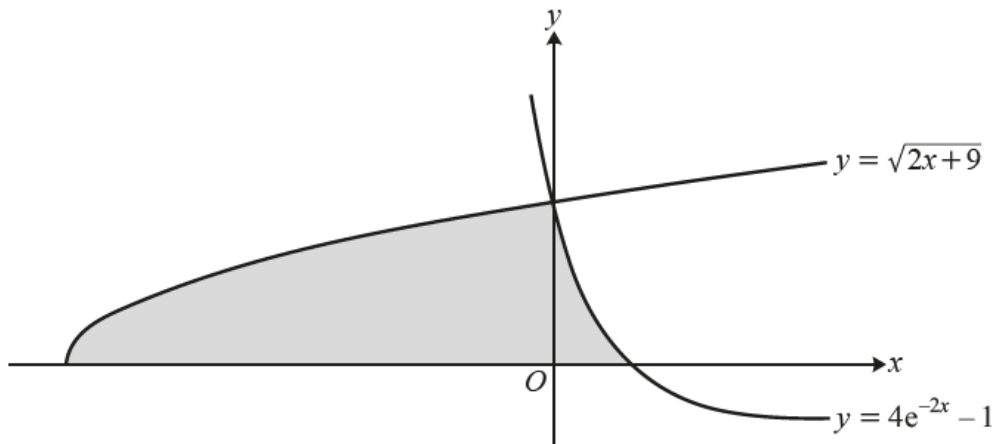
Determine the time taken for the tank to become empty.

[For a cone with base radius r and height h the volume V is given by $\frac{1}{3}\pi r^2 h$.]

[7]

5. June/2022/Paper_H240/03/No.6

In this question you must show detailed reasoning.



The diagram shows the curves $y = \sqrt{2x+9}$ and $y = 4e^{-2x} - 1$ which intersect on the y -axis. The shaded region is bounded by the curves and the x -axis.

Determine the area of the shaded region, giving your answer in the form $p + q \ln 2$ where p and q are constants to be determined.

[8]