Matrices – 2022 GCE AS Pure Further Mathematics A

1. June/2022/Paper_Y531/01/No.2

Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 5 \\ -1 & 0 \end{pmatrix}$ where a is a constant.

(a) Find the following matrices.

- $\mathbf{A} + \mathbf{B}$
- $\mathbf{AB}$
- $\mathbf{A}^2$

[3]

(b) **(i)** Given that the determinant of **A** is 25 find the value of a. **[2]**

(ii) You are given instead that the following system of equations does **not** have a unique solution.

$$ax + y = -2$$
$$-x + 3y = -6$$

Determine the value of a. **[2]**

2. June/2022/Paper_Y531/01/No.6

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \frac{1}{13}\begin{pmatrix} 5 & 12 \\ 12 & -5 \end{pmatrix}$.

You are given that $\mathbf{A}$ represents the transformation T which is a reflection in a certain straight line. You are also given that this straight line, the mirror line, passes through the origin, O.

(a) Explain why there must be a line of invariant points for T. State the geometric significance of this line. **[2]**

(b) By considering the line of invariant points for T, determine the equation of the mirror line. Give your answer in the form $y = mx + c$. **[4]**

The coordinates of the point P are $(1, 5)$.

(c) By considering the image of P under the transformation T, or otherwise, determine the coordinates of the point on the mirror line which is closest to P. **[3]**

(d) The line with equation $y = ax + 2$ is an invariant line for T.

Determine the value of a. **[2]**