Proof – 2022 GCE Pure Core 2 Further Math A Y541

1. June/2022/Paper_ Y541/01/No.7

You are given that a is a parameter which can take only real values.

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 4 & -6 \\ -3 & 10-4a & 9 \\ 7 & 4 & 4 \end{pmatrix}$.

(a) Find an expression for the determinant of $\mathbf{A}$ in terms of a. **[2]**

You are given the following system of equations in x, y and z.

$$\begin{aligned} 2x + \quad & 4y - 6z = 6 \\ -3x + \quad & (10-4a)y + 9z = -9 \\ 7x + \quad & 4y + 4z = 11 \end{aligned}$$

The system can be written in the form $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \\ 11 \end{pmatrix}$.

(b) (i) In the case where $\mathbf{A}$ is **not** singular, solve the given system of equations by using $\mathbf{A}^{-1}$. **[5]**

(ii) In the case where $\mathbf{A}$ **is** singular describe the configuration of the planes whose equations are the three equations of the system. **[3]**

The transformation represented by $\mathbf{A}$ is denoted by T.
A 3-D object of volume $|5a-20|$ is transformed by T to a 3-D image.

(c) (i) Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. **[1]**

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. **[2]**