Number Theory – 2022 GCE Additional Pure Further Math A Y545

1. June/2022/Paper_ Y545/01/No.2

Consider the integers a and b, where, for each integer n, $a = 7n + 4$ and $b = 8n + 5$.

Let $h = \text{hcf}(a, b)$.

(a) Determine all possible values of h. **[3]**

(b) Find all values of n for which a and b are **not** co-prime. **[2]**

2. June/2022/Paper_ Y545/01/No.4

Let N be the number $15\,824\,578$.

(a) **(i)** Use a standard divisibility test to show that N is a multiple of 11. **[2]**

(ii) A student uses the following test for divisibility by 7.

> 'Throw away' multiples of 7 that appear either individually or within a pair of consecutive digits of the test number.
> Stop when the number obtained is 0, 1, 2, 3, 4, 5 or 6.
> The test number is only divisible by 7 if that obtained number is 0.

For example, for the number N, they first 'throw away' the "7" in the tens column, leaving the number $N_1 = 15824508$. At the second stage, they 'throw away' the "14" from the left-hand pair of digits of N_1, leaving $N_2 = 01824\,508$; and so on, until a number is obtained which is 0, 1, 2, 3, 4, 5 or 6.

- Justify the validity of this process.
- Continue the student's test to show that $7 \mid N$. **[2]**

(iii) Given that $N = 11 \times 1\,438\,598$, explain why $7 \mid 1\,438\,598$. **[1]**

(b) Let $M = N^2$.

(i) Express N in the unique form $101a + b$ for positive integers a and b, with $0 \leqslant b < 101$. **[2]**

(ii) Hence write M in the form $M \equiv r \pmod{101}$, where $0 < r < 101$. **[1]**

(iii) Deduce the order of N modulo 101. **[1]**