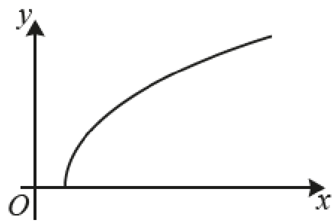
Numerical methods – 2022 GCE Pure Mathematics A

1. June/2022/Paper_H240/01/No.1



The diagram shows part of the curve $y = \sqrt{x^2 - 1}$.

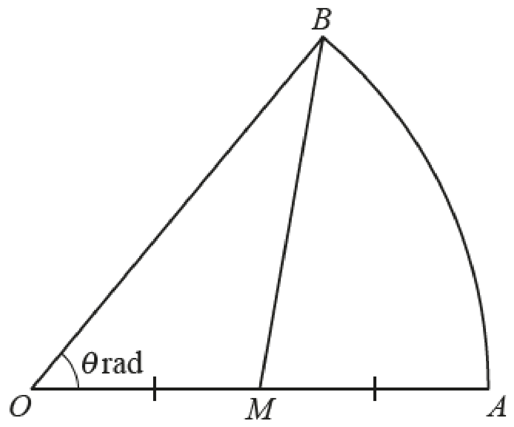
(a) Use the trapezium rule with 4 intervals to find an estimate for $\int_1^3 \sqrt{x^2 - 1}\,dx$.

Give your answer correct to **3** significant figures. **[4]**

(b) State whether the value from part **(a)** is an under-estimate or an over-estimate, giving a reason for your answer. **[1]**

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

2. June/2022/Paper_H240/01/No.10



The diagram shows a sector OAB of a circle with centre O and radius OA. The angle AOB is θ radians. M is the mid-point of OA. The ratio of areas $OMB : MAB$ is 2:3.

(a) Show that $\theta = 1.25 \sin\theta$. **[4]**

The equation $\theta = 1.25 \sin\theta$ has only one root for $\theta > 0$.

(b) This root can be found by using the iterative formula $\theta_{n+1} = 1.25 \sin\theta_n$ with a starting value of $\theta_1 = 0.5$.

- Write down the values of θ_2, θ_3 and θ_4.
- Hence find the value of this root correct to **3** significant figures. **[3]**

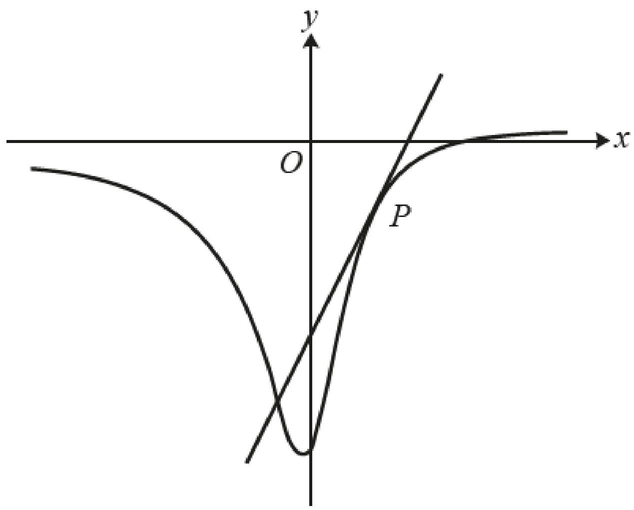
(c) The diagram in the Printed Answer Booklet shows the graph of $y = 1.25 \sin\theta$, for $0 \leqslant \theta \leqslant \pi$.

- Use this diagram to show how the iterative process used in **(b)** converges to this root.
- State the type of convergence. **[3]**

(d) Draw a suitable diagram to show why using an iterative process with the formula $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$ does not converge to the root found in **(b)**. **[2]**

3. June/2022/Paper_H240/03/No.5

In this question you must show detailed reasoning.



The diagram shows the curve with equation $y = \frac{2x-3}{4x^2+1}$. The tangent to the curve at the point P has gradient 2.

(a) Show that the x-coordinate of P satisfies the equation

$$4x^3 + 3x - 3 = 0.$$ **[5]**

(b) Show by calculation that the x-coordinate of P lies between 0.5 and 1. **[2]**

(c) Show that the iteration

$$x_{n+1} = \frac{3-4x_n^3}{3}$$

cannot converge to the x-coordinate of P whatever starting value is used. **[2]**

(d) Use the Newton-Raphson method, with initial value 0.5, to determine the coordinates of P correct to **5** decimal places. **[5]**