Sequences and Series – 2022 GCE AS Additional Pure Further Mathematics A

1. June/2022/Paper_Y535/01/No.3

   The sequence $\{U_n\}$ is given by $U_1 = 0$, $U_2 = -1$ and $U_{n+2} = U_{n+1} + U_n + n - 1$ for $n \geq 1$.

   (a) List the first seven terms of this sequence. [2]

   The Fibonacci sequence $\{F_n\}$ is given by $F_1 = 1$, $F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$.

   (b) (i) By comparing the two sequences, give the relationship between U_n and F_n. [2]

   (ii) Show that the relationship found in part (b)(i) holds for all $n \geq 1$. [3]

2. June/2022/Paper_Y535/01/No.6

The sequence $\{u_n\}$ is such that $u_1 = 7$, $u_2 = 37$, $u_3 = 337$, $u_4 = 3337$, ...

(a) Write down a first-order recurrence system for $\{u_n\}$. [2]

(b) By solving the recurrence system of part **(a)**, show that $u_n = \frac{1}{3}(10^n + 11)$. [5]

(c) Prove that $\{u_n\}$ contains infinitely many terms which are multiples of 37. [7]

2. June/2022/Paper_Y535/01/No.6

2