Sequences and Series – 2022 GCE Additional Pure Further Math A Y545

1. June/2022/Paper_ Y545/01/No.3

   Consider the integers a and b, where, for each integer n, $a = 7n + 4$ and $b = 8n + 5$.

   Let $h = \mathrm{hcf}(a, b)$.

   (a) Determine all possible values of h. [3]

   (b) Find all values of n for which a and b are **not** co-prime. [2]

2. June/2022/Paper_ Y545/01/No.6

In a national park, the number of adults of a given species is carefully monitored and controlled. The number of adults, n months after the start of this project, is A_n. Initially, there are 1000 adults. It is predicted that this number will have declined to 960 after one month.

The first model for the number of adults is that, from one month to the next, a fixed proportion of adults is lost. In order to maintain a fixed number of adults, the park managers "top up" the numbers by adding a constant number of adults from other parks at the end of each month.

(a) Use this model to express the number of adults as a first-order recurrence system. **[1]**

Instead, it is found that, the proportion of adults lost each month is double the predicted amount, with no change being made to the constant number of adults added each month.

(b) (i) Show that the revised recurrence system for A_n is $A_0 = 1000$, $A_{n+1} = 0.92A_n + 40$. **[1]**

    (ii) Solve this revised recurrence system. **[4]**

    (iii) Describe the long-term behaviour of the sequence $\{A_n\}$ in this case. **[1]**

A more refined model for the number of adults uses the second-order recurrence system $A_{n+1} = 0.9A_n - 0.1A_{n-1} + 50$, for $n \geqslant 1$, with $A_0 = 1000$ and $A_1 = 920$.

(c) (i) Determine the long-term behaviour of the sequence $\{A_n\}$ for this more refined model. **[4]**

    (ii) A criticism of this more refined model is that it does not take account of the fact that the number of adults must be an integer at all times.

       State a modified form of the second-order recurrence relation for this more refined model that will satisfy this requirement. **[1]**