Surfaces and Partial Differentiation – 2022 GCE AS Additional Pure Further Mathematics A

1. June/2022/Paper_Y535/01/No.2

The surface S has equation $z = x^3 + y^3 - 2x^2 - 5y^2 + 3xy$.

It is given that S has two stationary points; one at the origin, O, and the other at the point A.

Determine the coordinates of A. **[6]**

2. June/2022/Paper_Y535/01/No.5

A research student is using 3-D graph-plotting software to model a chain of volcanic islands in the Pacific Ocean. These islands appear above sea-level at regular intervals, (approximately) distributed along a straight line. Each island takes the form of a single peak; also, along the line of islands, the heights of these peaks decrease in size in an (approximately) regular fashion (see **Fig. 1.1**).

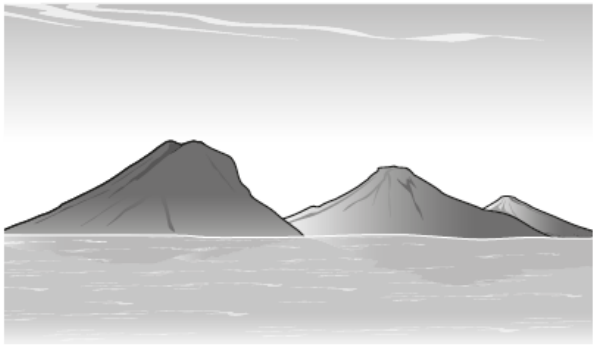

Fig. 1.1

The student's model uses the surface with equation $z = \sin x + \sin y$, a part of which is shown in **Fig. 1.2** below. The surface of the sea is taken to be the plane $z = 0$.

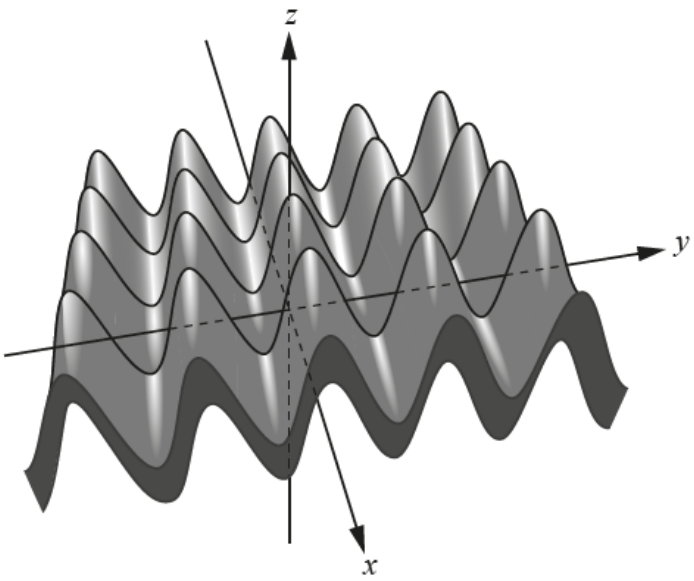

Fig. 1.2

(a) • Describe **two** problems with this model.
• Suggest revisions to this model so that each of these problems is addressed. **[4]**

(b) Still using their original model, the student examines the contour $z = 2$ for their surface only to find that the software shows what appears to be an empty graph.

Explain what has happened. **[2]**